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LETTER TO THE EDITOR

Another look at a good approximation of data for the distribution of COVID-19 in Cuba

Otra mirada hacia una correcta aproximación de datos para la distribución de la COVID-19 en Cuba

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ABSTRACT

In the article "Adjustment of population growth curve applied to Covid-19 in Cuba", the authors propose six types of models to approximate data from the distribution of COVID-19 in Cuba, while giving a clear answer to the possible advantages of some considerations. In this paper, we study intrinsic properties of some models of growth with polynomial variable transfer that give a very good approximation of the specific data on the pandemics in Cuba by June 5, 2020. The models have the right to exist in the treatment of issues from different fields of scientific knowledge. Numerical examples are presented using *CAS MATHEMATICA*.

Keywords: COVID-19, predictive study, population growth curves, Cuba.

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RESUMEN

En el artículo "Ajuste de curvas de crecimiento poblacional aplicadas a la COVID-19 en Cuba", los autores proponen y seriamente analizan seis tipos de modelos (logísticos y otros modelos de crecimiento) para aproximar los datos sobre la distribución de la COVID-19 en Cuba mientras plantean una clara respuesta a las posibles ventajas de algunas consideraciones. En este trabajo estudiamos las propiedades intrínsecas de algunos modelos de crecimiento con transferencia de variables polinómicas que proporcionan una muy buena aproximación de los datos específicos sobre la pandemia en Cuba antes del 5 de junio de 2020. Los modelos tienen el derecho a existir en el tratamiento de cuestiones de diferentes campos del conocimiento científico. Los ejemplos numéricos se presentan utilizando un modelo matemático para calcular los casos.

Palabras claves: COVID-19, estudio predictivo, curvas de crecimiento poblacional, Cuba.

Dear Editor:

In the article "Adjustment of population growth curve applied to Covid-19 in Cuba",(1) which was recently published in your journal, the authors propose six types of models to approximate data from the distribution of COVID-19 in Cuba.

In,(2,3) we define the following class of growth curves:

$$M(t) = A\frac{1-e^{-F(t)}}{1+e^{-F(t)}} \qquad [1]$$

where

$$F(t) = \sum_{i=0}^{n} a_i t^i;\ a_0 = 0. \qquad [2]$$

We will call this family the "Half–Logistic curve of growth with polynomial variable transfer" (HLCGPVT). The model corresponds to a certain kinetic reaction scheme that gave good results in the analysis of data from the spread of COVID-19 in Bulgaria and especially for predicting the expected "initial saturation level" at an early stage of forecasting.

In (4) Cordeiro, Alizadeh and Ortega proposed the following Exponential Half-Logistic Log-Logistic distribution with cumulative distribution function (cdf):

$$M_1(t) = \frac{1-\left(1+\left(\frac{t}{a}\right)^b\right)^{-\lambda}}{1+\left(1+\left(\frac{t}{a}\right)^b\right)^{-\lambda}}, \qquad [3]$$

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where $t > 0, a > 0, b > 0, \lambda > 0$.

In a particular case: $a = b = 1, \lambda > 0$, we have

$$M_1^*(t) = \frac{1-(1+t)^{-\lambda}}{1+(1+t)^{-\lambda}}. \qquad [4]$$

We define the following new class of growth curves with polynomial variable transfer:

$$M_1^{**}(t) = A\frac{1-(1+F(t))^{-\lambda}}{1+(1+F(t))^{-\lambda}} \qquad [5]$$

where

$$F(t) = \sum_{i=0}^{n} a_i t^i;\ a_0 = -1. \qquad [6]$$

In this article, we study some intrinsic properties of the models [1]–[2] and [5]–[6] with polynomial variable transfer that give a very good approximation of the specific data on the pandemic in Cuba by June 05, 2020.

Numerical experiments

The following data ”Total Coronavirus Cases in Cuba (10/03/2020 – 5/06/2020)” is used in the modelling process. (Figure 1).

Approximation with model [1]–[2]

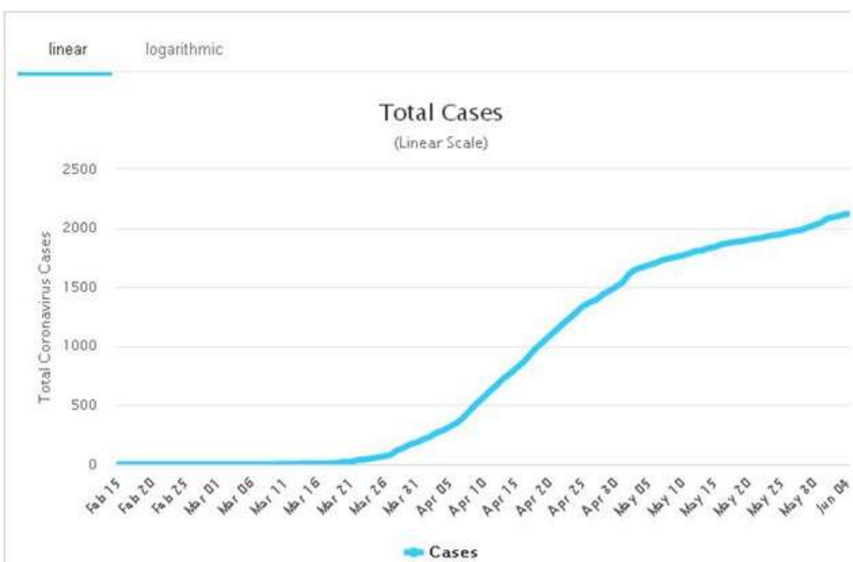

Fig. 1 - Total Coronavirus Cases in Cuba (10/03/2020 – 05/06/2020).

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For the real values in the specified period, the model – [1]–[2] for

$$n = 6, A = 2119, a_0 = 0, a_1 = -0.0653561, \quad a_4 = 0.030787, a_5 = -0.00397626,$$
$$a_2 = 0.123497, a_3 = -0.0898991, \quad a_6 = 0.000174555$$

is depicted on Fig. 2.b.

(We have adopted a scale on the horizontal axis: 0.1 division corresponds to 1 day).

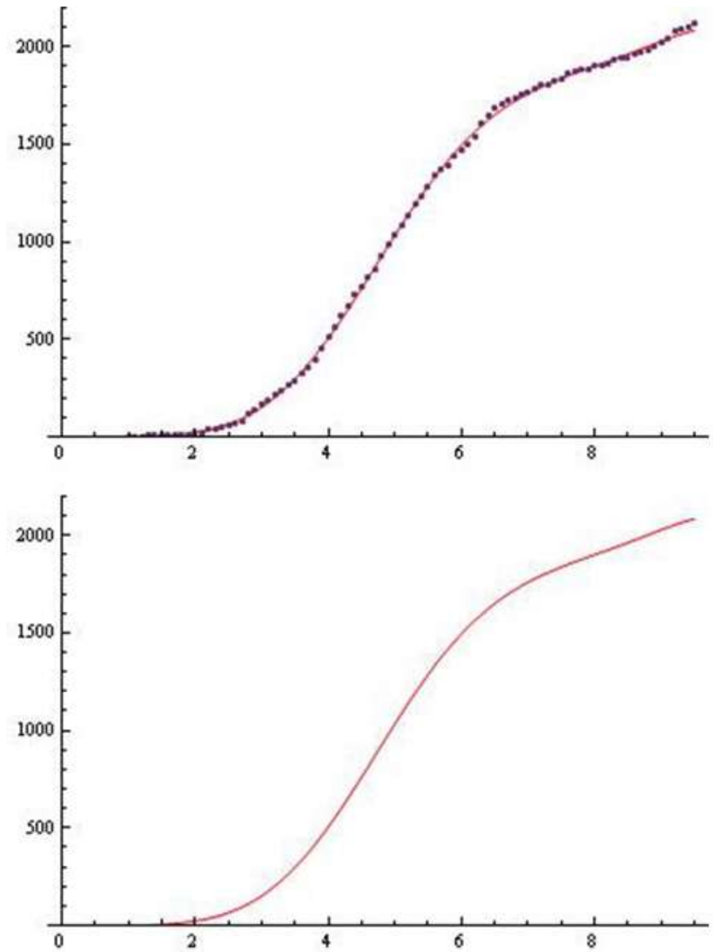

Fig. 2 - a) The "real values" (10/03/2020 – 05/06/2020); b) The model [1]–[2]

$$(n = 6, A = 2119, a_0 = 0, a_1 = -0.0653561,$$
$$a_2 = 0.123497, a_3 = -0.0898991,$$
$$a_4 = 0.030787, a_5 = -0.00397626,$$
$$a_6 = 0.000174555)$$

for the "real values" (10/03/2020 – 05/06/2020).

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Approximation with model [5]–[6]

For the data ”Total Coronavirus Cases in Cuba (10/03/2020 – 5/06/2020)” the model [5]–[6] for

$n = 9, A = 2119, \lambda = 1, a_0 = -1, a_1 = 4.51986,$

$a_2 = -7.92864, a_3 = 7.26067,$

$a_4 = -3.88988, a_5 = 1.28028, a_6 = 0.260823,$

$a_7 = 0.0319843, a_8 = -0.00215363,$

$a_9 = 0.0000609589$

is depicted on Fig. 3.b.

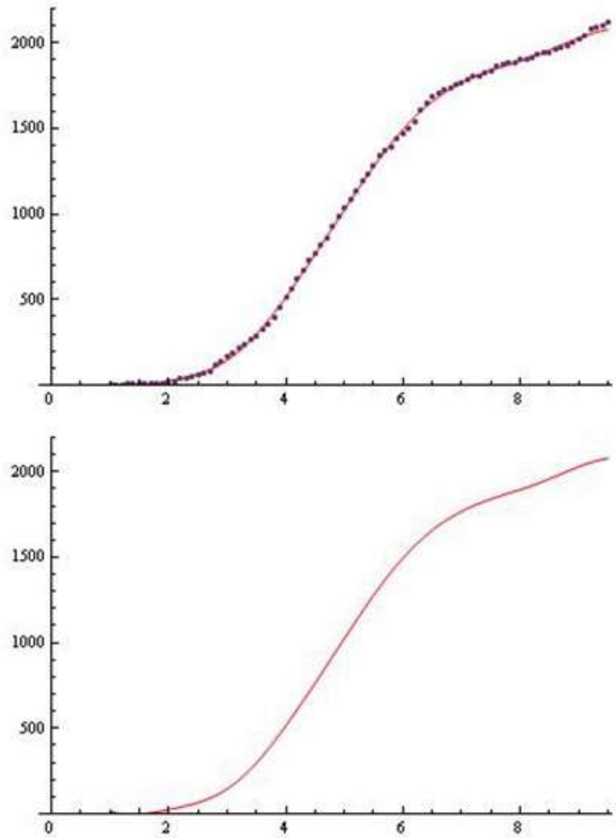

Fig. 3 - a) The "real values" (10/03/2020 – 05/06/2020); b) The model [5]–[6] ($n = 9, A = 2119, a_0 = -1, \lambda = 1, a_1 = 4.51986,$ $a_2 = -7.92864, a_3 = 7.26067, a_4 = -3.88988,$ $a_5 = 1.28028, a_6 = 0.260823, a_7 = 0.0319843,$ $a_8 = -0.00215363, a_9 = 0.0000609589$) for the "real values" (10/03/2020 – 05/06/2020).

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Concluding remarks

Appropriate parameters - $a_i; i = 1, 2, \ldots,$ for the various models of type [1]–[2] and [5]–[6] have been obtained using an optimization method for minimizing the sum of the squared differences between the data "real values" and the computed theoretical solution.

From Figures 2 and 3 we see that the approximations by the considered models (especially when approximating characteristic data in the specific section - at the end of the considered time interval) are very accurate.

In,[1] six models can be successfully expanded in the light of our considerations - by inserting corrective corrections of the type "polynomial variable transfer" and here we will skip their analysis.

It will take considerable time to propose adequate and powerful epidemiological models. We are convinced of this because, after such pandemics, there is always an "explosion" of serious scientific research!

In conclusion, we will explicitly note that the proposed research and future improvement of these models can be used successfully in the analysis of infection in the emerging and predicted by the World Health Organization next 1–3 scenarios of the disease.

We find similarities on the onset of the "initial saturation phase" for COVID-19 for Cuba and Bulgaria (approximate levels of pandemic control), which we believe is due to the extremely adequate sanitation and other measures taken by the governments and health authorities of both countries!

In addition, we are convinced (without being epidemiologists) that these achievements are due to the world-renowned successes of Cuban medical science and many years of experience in the prevention and treatment of lung diseases!

Appendix

For the ”saturation” by the model $M_1^*(t)$ to the horizontal asymptote $t = 1$ (i.e. $A = 1$) in Hausdorff sense[5] we have:

$$M_1^*(d) = \frac{1-(1+d)^{-\lambda}}{1+(1+d)^{-\lambda}} = 1-d \qquad [7]$$

or, equivalently,

$$H(d) := (1+d)^{\lambda} + 1 - \frac{2}{d} = 0. \qquad [8]$$

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The following theorem gives upper and lower bounds for d.

*Theorem***.** The ”saturation” - d satisfies the following inequalities for $\lambda \geq 1$:

$$d_l := \frac{2}{1+2\lambda} < d < \frac{2}{1+\sqrt{1+2\lambda}} := d_r. \qquad [9]$$

Proof. Clearly, $H'(t) > 0$ and $H(t)$ is an increasing function of $t \in [0,+\infty)$. Hence, if [8] it has a root, then it is unique. Using the inequality $(1+t)^{\lambda} > 1+\lambda t$ for $t > -1, \lambda > 1$ we have

$$H(d) > \frac{\lambda d^2 + 2d - 2}{d}.$$

The positive root of the quadratic in the numerator is

$$\frac{-1+\sqrt{1+2\lambda}}{\lambda} = \frac{2}{1+\sqrt{1+2\lambda}} = d_r.$$

Then

$$H(d_r) > 0. \qquad [10]$$

On the other hand, $H(d) < e^{\lambda d} + 1 - \frac{2}{d}$. Hence

$$H(d_l) = H\left(\frac{2}{1+2\lambda}\right) <$$

$$e^{\frac{2\lambda}{1+2\lambda}} + 1 - (1+2\lambda) = 2\lambda\left(\frac{e^{\frac{2\lambda}{1+2\lambda}}}{2\lambda} - 1\right).$$

For the derivation of

$$\eta(\lambda) = \frac{e^{\frac{2\lambda}{1+2\lambda}}}{2\lambda} - 1$$

we have

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$$\eta'(k) = -\frac{e^{\frac{2\lambda}{1+2\lambda}}}{2\lambda^2(1+2\lambda)^2}\left(4\lambda^2+2\lambda+1\right) < 0.$$

Therefore η is a decreasing function of λ. Using $\lambda > 1$ we have

$$H(d_l) < 2\lambda\eta(\lambda) <$$

$$2\lambda\eta(1) = 2\lambda\left(\frac{e^{\frac{2}{3}}}{2} - 1\right) < 0. \qquad [11]$$

Since H is an increasing function, the inequalities [10] and [11] imply that H has a unique root in the interval (d_l, d_r).

This completes the proof of the theorem.

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Conflict of interests

All authors declare no competing interests.

Contribution of authorship

All the authors participated in the discussion of the results and have read, reviewed and approved the final version of the article.

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